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A superlattice of 2d electrons in the fractional quantum Hall effect regime—the effects of tunnelling between different layers

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Abstract. The superlattice of parallel two-dimensional electron layers in a strong perpendicular magnetic field with small tunnelling between the layers is considered. For simple fractional filling factors in each layer we propose a new variational ground state. We study the collective excitation spectrum in single-mode approximation and make a conjecture on the offset of the fractional quantum Hall effect when the tunnelling becomes sufficiently strong.

1. Introduction

The fractional quantum Hall effect [1] (FQHE) is a remarkable phenomenon that arises from the Coulomb interaction between 2d electrons put into a strong perpendicular magnetic field, such that only the single Landau level is partially occupied. It is believed that it is a consequence of the dispersion relation of the intra-Landau-level collective excitations in the FQHE regime [2], which exhibits a gap at $k = 0$ and a magneto-roton minimum at the finite wavevector. Also, a restriction on the particle dynamics along the direction of the field, so that the system is effectively two-dimensional, appears to be essential for the effect. Recently [3, 4], interest has focused on superlattices of 2d electron layers in a strong magnetic field where a third degree of freedom, asociated with the motion of electrons along a field, can be introduced in a controlled manner. Theoretical studies of a free interacting electron gas [5, 6] as well as an interacting electron gas in a superlattice potential [7], in a strong uniform magnetic field, and all being done in the Hartree–Fock approximation, indicated that these systems have unexpectedly rich phase diagrams. However, the question of how to include the known tendency of electrons to form highly correlated uniform-density states in two dimensions, responsible to FQHE [8] has not been touched upon.

It has been shown that in a collective excitation spectrum of a superlattice of 2d electronic layers at filling $1/m$, with m-odd in each layer [9] and when tunnelling between layers is not allowed, the Coulomb interaction between electrons from different layers will introduce additional dispersion along the field, and that the magneto-roton minimum for low filling factors is most likely to collapse at the ends of the first Brillouin zone (at

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$k_z = \pi/c$, the z-axis is in the direction of the field, c is the distance between adjacent layers). In the present note we concentrate solely on the effect of electrons tunnelling from one layer to another in the FQHE regime in each layer. It is shown that the ground state has to change when a small hopping term is present in a Hamiltonian. A collective excitation spectrum calculated in single-mode approximation (SMA) will also be modified by this small perturbation, and it will be demonstrated that for hopping that is strong enough one should expect the system to go to a new phase.

2. The effects of electron tunnelling

We start by writing down the Hamiltonian for a system of parallel layers of electrons in a strong magnetic field with allowed hopping between nearest neighbouring layers and interaction between electrons from the same layer. In order to treat all electrons in our system as being indistinguishable the Hamiltonian is written in a second quantized form

$$\hat{H} = \sum_{i=1}^{N_l} \hat{H}_i - t \sum_{\langle i,j\rangle,\alpha} \hat{C}^+_{i,\alpha}\hat{C}_{j,\alpha} \tag{1}$$

where indices $\langle i, j\rangle$ denote the nearest neighbour layers, α denotes the single-particle states that span the lowest Landau level in a layer and $\hat{H}_i$ is a two-body Hamiltonian of interacting electrons from the same layer. We assume that only the lowest Landau level in each layer is partially occupied, the spins are completely polarized and we impose anticommutation relations

$$[\hat{C}^+_{i,\alpha}, \hat{C}_{j,\beta}]_+ = \delta_{i,j}\delta_{\alpha,\beta}\,[\hat{C}_{i,\alpha}, \hat{C}_{j,\beta}]_+ = 0. \tag{2}$$

Since we are interested in the spectrum of our Hamiltonian for sufficiently small tunnelling, first we must write down the ground state when $t = 0$. For simple filling factors $\upsilon = 1/m$, m-odd, this ground state should have Laughlin state correlations in each layer built in and also should be properly antisymmetrical with respect to all electrons. In the second quantized formalism this can be achieved by writing it as

$$|\Psi^\upsilon_0\rangle = \left(\prod_i \hat{L}^\upsilon_i\right)|0\rangle \tag{3}$$

where $|0\rangle$ is a vacuum and $\hat{L}^\upsilon_i$ is an operator which, acting on vacuum, creates a Laughlin state in a layer i. It is defined with

$$\hat{L}^\upsilon_i = \sum_{\alpha_1,\ldots,\alpha_N} A^\upsilon_{\alpha_1,\ldots,\alpha_N}\hat{C}^+_{i,\alpha_1}\ldots\hat{C}^+_{i,\alpha_N} \tag{4}$$

where N is a number of electrons per layer and coefficient $A^\upsilon_{\alpha_1\ldots\alpha_N}$ is defined as the overlap between the Laughlin wavefunction for filling factor υ and the Slater determinant of single particle states $\alpha_1, \alpha_2, \ldots \alpha_N$ (see [10]).

Now, we are in the position to be able to search for the state with a lower energy than that defined with (3) when $t \neq 0$ in the Hamiltonian (1) (but which is small compared with the particular energy defined later). We propose the new ground state of the following form

$$|\Phi\rangle = a(\exp \sum_{\langle i,j\rangle,\alpha} b_\alpha(i,j)\hat{C}^+_{i,\alpha}\hat{C}_{j,\alpha})|\Psi^\upsilon_o\rangle \tag{5}$$

where parameters a and b are to be determined variationally. To find the expectation

value of our Hamiltonian in this variational ground state as well as the norm of the state of the lowest order in t one assumes that coefficients $b_\alpha(i,j)$ are analytical in t and writes the state $|\Phi\rangle$ to the lowest order

$$|\Phi\rangle \approx (a + \sum_{\langle i,j\rangle,\alpha} b_\alpha(i,j)\hat{C}^+_{i,\alpha}\hat{C}_{j,\alpha})|\Psi^v_0\rangle. \tag{6}$$

Coefficients $b_\alpha(i,j)$ are symmetric in indices i and j and since we limit ourselves to a case of tunnelling only between nearest neighbours we may write them as dependent only on index α. The condition of state normalizability (6) is given by

$$|a|^2 + 2N_L \sum_\alpha |b_\alpha|^2\langle n_\alpha\rangle(1 - \langle n_\alpha\rangle) = 1 \tag{7}$$

where N_L is the number of layers and $\langle n_\alpha\rangle = \langle 0|\hat{L}^\dagger_i \hat{C}^+_{i,\alpha}\hat{C}_{i,\alpha}\hat{L}^v_i|0\rangle$ is the number of particles in state α averaged over the Laughlin state. Since the Laughlin state is a uniform isotropic liquid $\langle n_\alpha\rangle = v$. Introducing electron and hole excitation energies in the Laughlin state as

$$E_e(\alpha) = \langle 0|\hat{L}^{v+}_i \hat{C}_{i,\alpha}\hat{H}_i\hat{C}^+_{i,\alpha}\hat{L}^v_i|0\rangle \tag{8}$$

$$E_h(\alpha) = \langle 0|\hat{L}^{v+}_i \hat{C}^+_{i,\alpha}\hat{H}_i\hat{C}_{i,\alpha}\hat{L}^v_i|0\rangle \tag{9}$$

we can define $\Delta(\alpha) = E_e(\alpha) + E_h(\alpha) - 2E_0$ as the energy for creating an excitation above ground state, where $E_0 = \langle 0|\hat{L}^{v+}_i\hat{H}_i\hat{L}^v_i|0\rangle$ is just the energy of the Laughlin state for an isolated layer. Minimizing the expectation value $\langle\Phi|\hat{H}|\Phi\rangle$ with state $|\Phi\rangle$ being properly normalized to the lowest order in t, one obtains coefficients

$$b_\alpha = \frac{t}{\Delta(\alpha)} + O(t^3),\ \alpha = 1 + O(t^2) \tag{10}$$

and the ground state energy

$$\langle\Phi|\hat{H}|\Phi\rangle = N_L E_0 - 2N_L\frac{t^2}{\Delta} + O(t^4) \tag{11}$$

where now, new energy Δ is defined with $\Delta^{-1} = \Sigma_\alpha \Delta_\alpha^{-1}\langle n_\alpha\rangle(1 - \langle n_\alpha\rangle)$. Notice that Δ is always larger than zero, so state $|\Phi\rangle$ has a lower energy than a state $|\psi^v_0\rangle$. In this way we obtained the natural energy to compare with t; our calculation assumes the ratio $t/\Delta(\alpha)$ to be a small number.

Having established the ground state for our Hamiltonian we are in a position to study the spectrum of collective excitations using the projected SMA of Girvin *et al* [2]. In this approximation the energy of density fluctuations is given with

$$e(k,k_z) = f(k,k_z)/s(k,k_z) \tag{12}$$

and $f(k,k_z) = \langle\Phi|\hat{\rho}^+(k,k_z)[\hat{H},\hat{\rho}(k,k_z)]|\Phi\rangle$, and $s(k,k_z) = \langle\Phi|\hat{\rho}^+(k,k_z)\hat{\rho}(k,k_z)|\Phi\rangle$ is a static structure factor. The projected density operator in the second quantization is defined as

$$\hat{\rho}(k,k_z) = \int \exp[i(k\cdot r + k_z z)]\hat{\Psi}^+(r,z)\hat{\Psi}(r,z)\,dr\,dz \tag{13}$$

$$\hat{\Psi}(r,z) = \sum_{i,\alpha}\hat{C}_{i,\alpha}\Phi_\alpha(r)\xi(z - z_i) \tag{14}$$

where $k(r)$ and $k_z(z)$ are vectors in the plane and along the field, respectively, $\Phi(r)$ are

eigenfunctions of the eigen-momentum operator in symmetric gauge within the lowest Landau level, and $\zeta(z - z_i)$ is a Wannier function localized around layer i.

Since we are interested in $e(k, k_z)$ to the lowest order in t, the approximation of the projected density operator which would be correct for small tunnelling is needed at this point. We define $F_{\alpha,\beta}(k) = \int \exp(ik \cdot r)\Phi^*_\alpha(r)\Phi_\beta(r)\,dr$ and write the projected density operator in the form

$$\hat{\rho}(k, k_z) \approx \hat{\rho}_0(k, k_z) + \hat{\rho}_1(k, k_z) \tag{15}$$

where

$$\hat{\rho}_0(k, k_z) = \sum_{\alpha,\beta} F_{\alpha,\beta}(k) \sum_j \int \exp(ik_z z)\,|\zeta(z - z_j)|^2\,dz \hat{C}^+_{j,\alpha}\hat{C}_{j,\beta} \tag{16}$$

$$\hat{\rho}_1(k, k_z) = \sum_{\alpha,\beta} F_{\alpha,\beta}(k) \sum_{\langle i,j\rangle} \int \exp(ik_z z)\zeta(z - z_i)\zeta(z - z_j)\,dz \hat{C}^+_{i,\alpha}\hat{C}_{j,\beta} \tag{17}$$

We are actually taking the Wannier functions $\zeta(z)$ in a tight-binding approximation and assuming that only the overlap between nearest neighbours is important. In this approximation these functions are localized on a scale l (which in heterostructures happens to be of order of magnetic length) and is much smaller than the distance between adjacent plates c. Therefore, we will take in lowest order $\hat{\rho}_1 \sim l/c$. Also, $l/c \approx t/\Delta_1$ where Δ_1 is the energy of the order of magnitude of excitation $\Delta(\alpha)$. With all this in mind our projected density operators are written in the form

$$\hat{\rho}_0(k, k_z) = \sum_{j,\alpha,\beta} \exp(ik_z z_j)F_{\alpha,\beta}(k)\hat{C}^+_{j,\alpha}\hat{C}_{j,\beta} \tag{18}$$

$$\hat{\rho}_1(k, k_z) = (t/\Delta_1) \sum_{n=\pm1,j,\alpha,\beta} \exp(ik_z(z_j + nc/2))F_{\alpha,\beta}(k)\hat{C}^+_{j,\alpha}\hat{C}_{j+n,\beta} \tag{19}$$

with the additional assumption that important overlapping of tight-binding functions $\zeta(z)$ happens in the middle of two layers. If we insert the two last expressions in the definition of the projected static structure factor $s(k, k_z)$ it can be written as

$$s(k, k_z) = N_L s_0(k) + t^2(A(k)\cos(k_z c) + B(k)\cos(k_z c/2) + C(k)) \tag{20}$$

where

$$s_0(k) = N_L^{-1}\langle\Psi^v_0|\hat{\rho}^+_0\hat{\rho}_0|\Psi^v_0\rangle \tag{21}$$

is the projected static structure factor in the layer of Girvin *et al* and functions A, B and C are defined as

$$A(k) = (t^2\cos(k_z c))^{-1}\langle\Psi^v_0|\hat{T}\hat{\rho}^+_0\hat{\rho}_0\hat{T}|\Psi^v_0\rangle \tag{22}$$

$$B(k) = (t^2\cos(k_z c/2))^{-1}(\langle\Psi^v_0|\hat{T}\hat{\rho}^+_0\hat{\rho}_1|\Psi^v_0\rangle + \text{complex conjugate}) \tag{23}$$

and

$$C(k) = t^{-2}\langle\Psi^v_0|\hat{\rho}^+_1\hat{\rho}_1|\Psi^v_0\rangle \tag{24}$$

and $\hat{T}$ stands for $\Sigma_{n=\pm1,i,\alpha}(t/\Delta(\alpha))\hat{C}^+_{i,\alpha}\hat{C}_{i+n,\alpha}$. It is straightforward but tedious to show that functions A, B and C are functions of k and not of k_z. Also, we define them not to be functions of t either, so the first correction in the static structure factor (20) to the case without hopping is of the order of t^2.

To obtain the energy of collective excitations (14) one has to calculate $f(\boldsymbol{k}, k_z)$. In our approximation this can be written as

$$f(\boldsymbol{k}, k_z) = N_L f_0(\boldsymbol{k}) + t^2[D(\boldsymbol{k})\cos(k_z c) + E(\boldsymbol{k})\cos(k_z c/2) + F(\boldsymbol{k})] \tag{25}$$

where

$$f_0(\boldsymbol{k}) = N_L^{-1}\langle\Psi_0^g|\hat{\rho}_0^+[\hat{H}_0, \hat{\rho}_0]|\Psi_0^g\rangle \tag{26}$$

is again the in-the-layer projected $f(\boldsymbol{k})$ function [2] and

$$D(\boldsymbol{k}) = (t^2\cos(k_z c))^{-1}\langle\Psi_0^g|\hat{\rho}_0^+[\hat{H}_0, \hat{\rho}_0]\hat{T} + \hat{T}^+\hat{\rho}_0^+[\hat{H}_1, \hat{\rho}_0] + \hat{T}^+\hat{\rho}[\hat{H}_0, \hat{\rho}_0]\hat{T}|\Psi_0^g\rangle \tag{27}$$

$$E(\boldsymbol{k}) = (t^2\cos(k_z c/2))^{-1}\langle\Psi_0^g|\hat{\rho}_0^+[\hat{H}_1, \hat{\rho}_1] + \hat{\rho}_1^+[\hat{H}_1, \hat{\rho}_0] + \hat{\rho}_0^+[\hat{H}_0, \hat{\rho}_1]\hat{T} + \hat{\rho}_1^+[\hat{H}_0, \hat{\rho}_0]\hat{T} + \hat{T}^+\hat{\rho}_0^+[\hat{H}_0, \hat{\rho}_1] + \hat{T}^+\hat{\rho}_1^+[\hat{H}_0, \hat{\rho}_0]|\Psi_0^g\rangle \tag{28}$$

$$F(\boldsymbol{k}) = t^{-2}\langle\Psi_0^g|\hat{\rho}_1^+[\hat{H}_0, \hat{\rho}_1]|\Psi_0^g\rangle \tag{29}$$

where we abbreviated $\hat{H}_0 = \Sigma_i^{N_L}\hat{H}_i$ and $\hat{H}_1 = -t\Sigma_{\langle i,j\rangle,\alpha}\hat{C}_{i,\alpha}^+\hat{C}_{j,\alpha}$. Functions D, E and F again depend only on $\boldsymbol{k}$, not on t or k_z and have the dimension [energy]$^{-1}$.

Using (20) and (25) we can write $e(k, k_z)$ to the lowest order in tunnelling t

$$e(\boldsymbol{k}, k_z) = e_0(\boldsymbol{K})[1 - t^2/(N_L s_0(\boldsymbol{K}))((A(\boldsymbol{k}) - D(\boldsymbol{k})/e_0(\boldsymbol{k}))\cos(k_z c) + (B(\boldsymbol{k}) - E(\boldsymbol{k})/e_0(\boldsymbol{k}))\cos(k_z c/2) + C(\boldsymbol{k}) - F(\boldsymbol{k})/e_0(\boldsymbol{k}))] \tag{30}$$

where $e_0(\boldsymbol{k})$ is the dispersion relation of the collective excitation spectrum for a 2d layer of interacting electrons [2].

To calculate dispersion (30) explicitly one gets involved in enormous algebraic difficulties and in intrinsic problems related to representing the Laughlin wavefunction in the second quantization. We are currently involved in such considerations but it turns out that this information is not essential for the following discussion, and we will concentrate on the magneto-roton minimum in our spectrum of collective excitations. For small t the minimum of function $e(\boldsymbol{k}, k_z)$ will occur at $|\boldsymbol{k}| \approx 1/a$, a being the average distance between particles in the same plane, because in (30) the second term in the square bracket is much less than one. To see where this minimum will show up on the k_z-axis one should notice that the presence of the $\cos(k_z c/2)$ term in (30) is an artefact of our approximation that the important overlap of functions $\zeta(z)$ in a tight-binding picture takes place at a half distance between plates. If the most significant overlapping is at some other point, then in our approximation instead of having $c/2$ in the final formula for $e(\boldsymbol{k}, k_z)$, we would have some other fraction of c. Since the position of the minimum of excitation spectrum on the k_z-axis depends only on the symmetry, and is insensitive to details of our approximation, the only value of k_z for which simultaneously $\cos(k_z c)$ and $\cos(k_z c/q)$, $q > 1$, reach their extremums, for every q that we choose, is $k_z = 0$ (or, equivalently $k_z = 2\pi/c$). The introduction of a third degree of freedom along the field via small tunnelling of electrons from one layer to another in Hamiltonian (1) certainly cannot enhance the gap in the spectrum so we write a magneto-roton minimum of $e(\boldsymbol{k}, k_z)$ in the presence of small tunnelling

$$\Delta_{\mathrm{mr}} = \Delta_{\mathrm{mr}}^0(1 - \mathrm{const}(t/\Delta_{\mathrm{mr}}^0)^2) \tag{31}$$

where Δ_{mr}^0 is the energy of the magneto-roton minimum for a purely 2d system of [2].

Equation (31) is valid in the limit of $t/\Delta(\alpha) \ll 1$ with a constant of order unity. Therefore, we suppose that for some critical value of tunnelling energy t_c, which is of the order of the Laughlin excitation energy, the gap in the spectrum collapses and the system becomes unstable towards a formation of a Wigner crystal, as is expected in a crossover from a 2d to 3d interacting electron gas in strong magnetic field [11]. Since, in our Hamiltonian, interaction between electrons from different layers is not included, the density of the electrons in this new phase has to be periodic along the z-axis with period c, the opposite to the case with this interaction but without hopping [9], when one expects a body-centred-tetragonal (or hexagonal) lattice to be formed (period $2c$). Therefore, on physical grounds, one should also expect a minimum of $e(k, k_z)$ to occur at $k_z = 0$ (or, equivalently at $2\pi/c$).

3. Conclusion

As a conclusion, we have demonstrated that when small tunnelling between 2d layers of electrons is present in the superlattice, both the ground state and the energy of collective excitations is changed. For much smaller tunnelling than the energy of the Laughlin quasiparticle excitations, at simple filling factors $\nu = 1/m$ FQHE still occurs. However, we have demonstrated that the energy of the magneto-roton minimum is renormalized by the presence of a hopping term in the Hamiltonian and we argue that for strong enough tunnelling a Wigner crystal is formed in each layer, and the FQHE is ultimately destroyed.

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